South Central College

MATH 233 Multivariable Calculus

Course Outcome Summary

Course Information

Description Multivariable Calculus extends the notions of Calculus I and Calculus II to functions of more than one variable. Topics include such things as curves and surfaces in Euclidean n-space, partial derivatives, directional derivatives, tangent plans and differentials, double- and triple-integrals, the rectangular, cylindrical and spherical coordinate systems, line integrals, surface integrals, Green's theorem, Stokes' theorem and the divergence theorem.

(MNTC goal area: 4); (prerequisite - MATH 132 with a grade of C or better)

Total Credits 4

Total Hours 80

Types of Instruction

Instruction Type	Credits/Hours
Lecture	3/48
Lab	1/32

Pre/Corequisites

MATH 132 with a grade of C or better.

Institutional Core Competencies

Critical and Creative Thinking - Students will be able to demonstrate purposeful thinking with the goal of using a creative process for developing and building upon ideas and/or the goal of using a critical process for the analyzing and evaluating of ideas.

Course Competencies

1. **Describe objects in the three-dimensional rectangular coordinate system**

   Learning Objectives
   Define function and associated terms precisely
   Model linear behavior with linear functions
   Model growth and decay behavior with exponential and logarithmic functions
   Model circular and cyclic behavior with the trigonometric functions

2. **Explain two- and three-dimensional vector operations operations**

3. **Apply the dot product to vector operations**

Learning Objectives
Graph functions using transformations
Use symmetry properties to expedite graphing
Parameterize plane curves

4. **Apply the cross product to vector operations**

Learning Objectives
Deduce properties of inverse functions in general
Parameterize inverse functions
Explain the behavior of the logarithmic functions
Explain the behavior of the inverse trigonometric functions

5. **Describe lines and planes in space**

Learning Objectives
Apply increments to problems concerning constant rates of change
Approximate change in a variable which varies continuously, using secant lines
Interpret a limit geometrically as constraining a function close to a certain value
Define limit precisely using the delta-epsilon notation
Demonstrate limiting behavior of common functions

6. **Explain the behavior of vector-valued functionsa.Define a vector-valued functionb.Define the limit of a vector-valued functionc.Define continuity of a vector-valued functiond.Define the derivative of a vector-valued functione.Interpret velocity, direction, speed and acceleration in terms of vector-valued functions**

Learning Objectives
Deduce rules for easily finding limits of certain algebraic functions
Define one-sided limits of a function
Compute one-sided limits of a function

7. **Explain the behavior of vector-valued functionsa.Define a vector-valued functionb.Define the limit of a vector-valued functionc.Define continuity of a vector-valued functiond.Define the derivative of a vector-valued functione.Interpret velocity, direction, speed and acceleration in terms of vector-valued functions**

Learning Objectives
Determine where functions may grow without bound near a point
Compute limits of functions when domain values grow or decrease without bound
Explain how a function may fail to have a limit at a certain point in the domain

8. **Extend the calculus to vector-valued functionsa.Derive differentiation rules for vector-valued functionsb.Define the indefinite integral of a vector-valued functionc.Define the definite integral of a vector-valued function**

Learning Objectives
Interpret the Intermediate Value Theorem geometrically
Apply the Intermediate Value Theorem to root finding

9. **Interpret length of a smooth curvea.Compute the length of arcb.Compute speed on a smooth curvec.Define the unit tangent vector**

Learning Objectives
Prove a certain function is continuous at a point in its domain
Prove a certain function is continuous everywhere
Invent a counterexample showing discontinuity of a function at a point in its domain
Define continuity precisely using the delta-epsilon notation
Derive properties of continuous functions from the definition

10. **Explain curvaturea.Define curvature preciselyb.Derive a formula for computing curvaturec.Define the principal unit normal vectord.Derive a formula for computing the unit**

normal vectore.Extend curvature and normal vectors to space curvesf.Define torsion

Learning Objectives
Interpret the derivative geometrically in terms of a tangent line
Interpret the derivative as an instantaneous rate of change
Define the derivative as the limit of a difference quotient
Define the one-sided derivative
Explain the connection between differentiability and continuity
Extend the definition to higher order derivatives

11. Explore real-valued functions of more than one variablea.Define a function of n independent variablesb.Explain interior and boundary pointsc.Explain open and closed setsd.Explain bounded and unbounded regions in the plane

Learning Objectives
Show that the derivative is a linear operator
Compute the derivative of a polynomial
Extend differentiation rules to include negative exponents

12. Graph functions of more than one variablea.Depict a surface algebraicallyb.Illustrate graph behavior by means of level curvesc.Extend b, above, to the notion of level surfaced.Define interior and boundary points for space regions

Learning Objectives
Compute the derivative of a sum of two functions
Compute the derivative of the difference of two functions
Compute the derivative of the product of two functions
Compute the derivative of the quotient of two functions
Compute the derivative of composite functions using the chain rule

13. Extend the ideas of limits and continuity to higher dimensionsa.Define the limit of a function of two variablesb.Intuit properties of limits of functions of two variablesc.Define continuity for functions of two variables

Learning Objectives
Derive the limit formulas for expressions containing sines and cosines
Derive formulas for the derivatives of sine and cosine
Extend the formulas from (b), above, to the remaining trigonometric functions

14. Define partial derivativea.Define the partial with respect to xb.Define the partial with respect to y c.Extend these ideas to functions of more than two variablesd.Explain the connection between partial derivatives and continuitye.Explore Clairaut's Theoremf.State the connection between differentiability and continuity

Learning Objectives
Differentiate functions defined by parametric equations
Differentiate functions defined implicitly
Apply differentiation to situations modeled by related rates

15. Prove the Chain Rule for partial derivativesa.Derive the Chain Rule for functions of two variablesb.Derive the Chain Rule for functions of three variablesc.Apply the Chain Rule to physical problems

Learning Objectives
Prove Rolle□s Theorem
Prove the Mean Value Theorem for Derivatives
Derive practical results from the Mean Value Theorem for Derivatives
Interpret the Mean Value Theorem for Derivatives graphically

16. Implement the directional derivativea.Define the directional derivativeb.Interpret the directional derivativec.Compute the directional derivatived.Define the gradient vectore.Illustrate algebraic rules of the gradient vectorf.Show the directional derivative is a dot productg.Explore properties of the directional derivative

Learning Objectives

Explain the Intermediate Value Theorem
Use the first derivative test to locate intervals of increasing or decreasing behavior
Use the first and second derivatives to locate local extrema of a function
Use the second derivative to determine the concavity of the graph of a function
Use the second derivative to locate points of inflection on the graph of a function

17. Determine tangent planesa.Define tangent planeb.Define normal linec.Find the plane tangent to a pointd.Find the line normal to a pointe.Find the plane tangent to a surface

Learning Objectives

Apply differentiation to problems from business and economics
Apply differentiation to problems from the manufacturing industries
Apply differentiation to problems from mathematics, physics, optics and mechanics

18. Linearize a functiona.Use the differential to estimate change in a directionb.Explain linearization and the standard linear approximationa.Define the total differential

Learning Objectives

Define differential in terms of the derivative
Estimate rate of change with the differential
Approximate a function□s local behavior using a linear expression

19. Find extreme pointsa.Define local maximumb.Define local minimumc.Give the first derivative test for local extreme valuesd.Define critical pointe.Define saddle pointf.Explain the second derivative test for local extreme valuesg.Extend the above to the problem of absolute extrema on closed bounded regions

Learning Objectives

Define antiderivative as an inverse operator
Derive rules for antiderivatives from those of derivatives
Apply antiderivatives to initial value problems and simple exact differential equations

20. Develop the properties of double integralsa.Render a double integral over a rectangular regionb.Interpret a double integral as a volumec.Calculate a double integral via Fubini's Theoremd.Compute a double integral over certain bounded non-rectangular regionse.Find limits of integration

Learning Objectives

Demonstrate that the antiderivative is a linear operator
Compute an antiderivative using the power rule in integral form
Compute an antiderivative using substitution

21. Apply double integrals to real-world problemsWhat you will learn as you master this competency:a.Find the area of a bounded region in the planeb.Find the average value of a functionc.Find the moment of a thin flat plated.Find the center of mass of a thin flat plate

Learning Objectives

Interpret expressions containing the sigma notation
Define the definite integral as the limit of a Riemann sum
Interpret area under a curve as a definite integral
Define average value as a definite integral

22. Interpret double integrals in polar formWhat you will learn as you master this competency:a.Describe the double integral in polar formb.Find the limits of integrationc.Determine area in polar coordinatesd.Relate polar and Cartesian integrals

Learning Objectives

Demonstrate linearity for definite integrals
Demonstrate sign reversal when the order of integration is reversed
Explain the Mean Value Theorem for Integrals
Show max-min bounds for a definite integral

23. **Extend the calculus to triple integrals in rectangular coordinatesWhat you will learn as you master this competency:a.Define volume as a triple integralb.Find the limits of integrationc.Find the average value of a function in spaced.Compute a mass in three dimensionse.Compute a moment in three dimensions**

    Learning Objectives
    Differentiate a definite integral
    Evaluate a definite integral by means of an indefinite integral

24. **Calculate triple integrals in other coordinate systemsWhat you will learn as you master this competency:a.Integrate in cylindrical coordinatesb.Relate rectangular to cylindrical coordinatesc.Describe the spherical coordinate systemd.Relate rectangular to spherical coordinatese.Integrate in spherical coordinatesf.Convert between the various coordinate systems**

    Learning Objectives
    Use substitution without changing the limits of integration
    Use substitution while changing the limits of integration

25. **Define the line integral over a curveWhat you will learn as you master this competency:a.Evaluate a line integralb.Explore useful properties of line integralsc.Interpret mass, moment and other properties in terms of line integrals**

    Learning Objectives
    Compute the area under a curve
    Compute the area between two curves

26. **Depict vector fieldsWhat you will learn as you master this competency:a.Define a gradient fieldb.Define work done over a smooth curvec.Evaluate a work integral**
27. **Explore special instances of vector fieldsWhat you will learn as you master this competency:a.Define path independenceb.Prove the Fundamental Theorem of Line Integralsc.Apply a and b, above, to problems involving work**
28. **Explain exact differential formsWhat you will learn as you master this competency:a.Define exact differential formb.Give the component test for exactnessc.Show how to prove that a differential form is exact**
29. **Apply Green's Theorem in the PlaneWhat you will learn as you master this competency:a.Define divergenceb.Define the k-component of curlc.Explain the normal form of Green's Theoremd.Explain the tangential form of Green's Theoreme.Prove Green's theorem for certain special regions**
30. **Solve problems involving surface integralsWhat you will learn as you master this competency:a.Explain the formula for surface area as a double integralb.Define surface integralc.Define surface area differentiald.Define fluxe.Find mass and moments of thin shells**
31. **Parameterize a surfaceWhat you will learn as you master this competency:a.Parameterize various common surfacesb.Define a smooth parameterized surfacec.Define area of a smooth surfaced.Explain a parametric surface integral**
32. **Apply Stokes' TheoremWhat you will learn as you master this competency:a.Explain Stokes' Theoremb.Prove Stokes' Theorem for certain surfacesc.Show the relationship of curl and grad**
33. **Apply the Divergence TheoremWhat you will learn as you master this competency:a.Define divergence of a vector field in three dimensionsb.Prove the Divergence Theorem for certain regions**

SCC Accessibility Statement

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