South Central College

MATH 240 Elementary Linear Algebra

Course Outcome Summary

Course Information

Description This is a first course in linear algebra for students intending to go into engineering, mathematics, the sciences, economics, statistics and other technical fields. Among the topics covered are matrices, determinants, systems of linear equations, vector spaces, linear transformations and characteristic value problems. Apart from the useful and practical topics considered, the course also serves as an introduction to the notion of mathematical proof.

Total Credits 4

Total Hours 64

Types of Instruction

Instruction Type	Credits/Hours
Lecture	4

Institutional Core Competencies

Critical and Creative Thinking - Students will be able to demonstrate purposeful thinking with the goal of using a creative process for developing and building upon ideas and/or the goal of using a critical process for the analyzing and evaluating of ideas.

Course Competencies

1. Solve systems of linear equations

Learning Objectives
Understand the language of systems of linear equations
Recognize the operations that lead to equivalent systems

2. Depict a system of equations as a matrix and solve

Learning Objectives
Define matrix
Use the elementary row operations to transform an augmented matrix to row echelon form
Implement the Gauss-Jordan method to mechanize this process
Note the properties of the special case of homogeneous systems

3. Apply systems of linear equations to other areas

Learning Objectives
Fit a polynomial curve to a desired set of points

Apply systems of linear equations to network analysis

4. Develop a matrix algebra

Learning Objectives
Define equality of matrices
Define matrix addition
Define scalar multiplication of a matrix
Define matrix multiplication
Use partitioning to simplify problems

5. Prove various properties of matrix addition, scalar multiplication and matrix multiplication

Learning Objectives
Prove commutative properties
Prove associative properties
Prove distributive properties
Prove the properties of the zero matrix
Prove the associative and distributive properties of matrix multiplication
Prove the properties of the identity matrix

6. Classify the number of solutions of a system

Learning Objectives
Establish a sufficient condition for a unique solution
Establish a sufficient condition for an infinite number of solutions
Establish a sufficient condition for no solutions

7. Develop properties of the transpose matrix

Learning Objectives
Define the transpose of a matrix
Find the transpose of a transpose
Find the transpose of a sum
Find the transpose of a scalar multiple
Find the transpose of a product of two matrices

8. Define the inverse of a matrix and explain its properties

Learning Objectives
Define the inverse of an invertible matrix
Prove that the inverse of a matrix is unique
Find the inverse of a matrix using Gauss-Jordan elimination
Find the inverse of a matrix product
Prove the left-right cancellation properties for matrix equations
Establish a sufficient condition for a unique solution in terms of invertibility

9. Build the rules for manipulating elementary matrices

Learning Objectives
Represent the elementary row operations as a matrix product of elementary matrices
Define row equivalence
Prove that elementary matrices are invertible
Establish a sufficient condition for invertibility in terms of elementary matrices
Define LU-Factorization

10. Derive properties of determinants

Learning Objectives
Define the determinant of a two-by-two matrix
Define minor
Define cofactor
Generalize the definition of determinant to larger matrices
Prove the pleasant property of determinants of triangular matrices

11. Simplify methods for computing determinants

Learning Objectives

Deduce rules for exploiting elementary row operations
Show that these rules can be phrased in terms of column operations
Prove conditions that lead to a zero determinant
Find the determinant of a matrix product
Find the determinant of a scalar multiple of a matrix
Find the necessary and sufficient conditions for invertibility in terms of determinants
Compute the determinant of an inverse matrix
Compute the determinant of a transpose matrix

12. Apply determinants to other areas

Learning Objectives

Define the adjoint of a matrix
Show how to compute the inverse of a matrix in terms of its adjoint
Prove Cramer's Rule
Compute certain areas and volumes via determinants
Test for collinear points in the plane
Exhibit the two-point form of an equation of a line
Exhibit the three-point form of an equation of a plane

13. Define vector

Learning Objectives

Define a vector in the plane
Prove various properties of vector addition and scalar multiplication
Extend these ideas to vectors in Rn

14. Define vector space

Learning Objectives

Explain the ten axioms necessary for a vector space
Overview several common and important vector spaces
Prove additional properties of scalar multiplication

15. Establish a test for a subspace

Learning Objectives

Define a subspace of a vector space
Prove a sufficient condition for a subspace
Show that the intersection of subspaces is a subspace itself

16. Connect the ideas of spanning sets and linear independence

Learning Objectives

Define linear combination
Define a spanning set
Show that the span(S) is a subspace
Define linear dependence and independence
Test for linear dependence or independence

17. Describe a vector space in terms of basis and dimension

Learning Objectives

Define basis
Establish uniqueness of basis representation
Show the connection between the number of members in a set and linear dependence
Define the dimension of a vector space
Show tests for a basis in an n-dimensional space

18. Apply the idea of rank to a system of linear equations

Learning Objectives

Define row-space and column-space
Establish properties of row-equivalent matrices

Find the basis for the row space of a matrix
Prove the equality of dimension of row- and column-spaces
Define the rank of a matrix
Derive the special subspace of Rn called the null space
Apply these ideas to determine the solutions of a nonhomogeneous linear system

19. Show the connection between coordinates and change of basis

Learning Objectives

Define coordinate representation relative to a basis
Illustrate coordinate representation in Rn
Show the change of basis in Rn
Apply these ideas to the conic sections and rotation

20. Explain the dot product

Learning Objectives

Define length of a vector
Derive some rules concerning lengths
Define the unit vector
Define direction
Particularize these to the dot product and angle between vectors in the plane
Prove properties of the dot product
Prove the Cauchy-Schwarz Inequality
Prove the Triangle Inequality
Define orthogonal vectors

21. Generalize the notion of dot product to inner product

Learning Objectives

Define an inner product
Illustrate several common examples
Prove various properties of inner products
Define norm, distance and angle
Create orthogonal projections in inner product spaces
Relate orthogonal projections and distance
Extend to orthonormal bases
Explain the Gram-Schmidt orthonormalization process

22. Define and use the cross product

Learning Objectives

Define the cross product of two vectors
Derive algebraic properties of the cross product
Derive geometric properties of the cross product

23. Explain linear transformations

Learning Objectives

Define linear transformation
Derive certain properties of linear transformations
Find the linear transformation given by a matrix

24. Relate the kernel and range of a linear transformation

Learning Objectives

Define kernel of a linear transformation
Prove several theorems concerning the kernel
Explore the range of a linear transformation
Depict one-to-one and onto linear transformations
Define isomorphism

25. Explain linear transformations in terms of matrices

Learning Objectives

Find the standard matrix of a linear transformation

Explain the composition of linear transformations
Show the requirements for the existence of an inverse linear transformation
Define similar matrices
Prove properties of similar matrices

26. Explore the attributes of eigenvalues, eigenvectors and eigenspaces

Learning Objectives
Define eigenvalue and eigenvector
Show how to find these for a particular matrix
Show how to find these in terms of a linear transformation
Diagonalize a matrix
Relate eigenvalues to the notion of similar matrices
Prove the condition for diagonalization

27. Explain the ideas leading to the Spectral Theorem

Learning Objectives
Define symmetric matrix
Explain the meaning of the Spectral Theorem applied to the reals
Define an orthogonal matrix
Derive some properties of orthogonal and symmetric matrices

SCC Accessibility Statement

South Central College strives to make all learning experiences as accessible as possible. If you have a disability and need accommodations for access to this class, contact the Academic Support Center to request and discuss accommodations. North Mankato: Room B-132, (507) 389-7222; Faribault: Room A-116, (507) 332-7222.

Additional information and forms can be found at: www.southcentral.edu/disability

This material can be made available in alternative formats by contacting the Academic Support Center at 507-389-7222.